Record hot years in autocorrelated series

We want to know the probability that among 129 annual temperature readings, at least 15 of the hottest years appear in the last 19 places (1990-2008), much like they did in the reality

```
(* if there were cooling, we could be equally concerned,
so we will flip the sign of a series to guarantee
 that there was a net warming in those 129 years.

  If you're interested in the condition requiring
 strict "hottest" years in the recent decades,
discarding the cases of "coldest" years, you should
 approximately divide the resulting probability by two. *)
```

Run Evaluation / Evaluate notebook

```
planets = 5000; (* how many
 random sequences we analyze *)

(* Generating a random sequence
 of 129 temperature readings *)
(* First, the universal constants *)

{t, recordYearsLookedFor,
  whereAllowed, damping} =
 {127, 13, 17, 0.99}
(* the total number of years;
how many record breakers we looked
 for; and in how many recent years
 they are required to appear *)

(* Von Storch et al. have
 {127,13,17} but two years later,
you can change it to {129,15,19} *)
```

```
alarmistPlanets = 0;
 (* it counts the planets where
 the condition is satisfied *)

For [j = 1, j ≤ planets, j++,

  a = Table[0, {i, t}]; b = a; c = a;
  (* a[[1]] is 1880, assumed to have
   temperature anomaly of zero *)

  For [i = 2, i ≤ t, i++,
   a[[i]] = a[[i - 1]] +
     (RandomReal[] - 0.5) * 1;
    (* This adds a Brownian motion-
     like noise to the previous year,
   to derive the new temperature.
     The overall normalization,
   determined by "times one" at
    the end, doesn't influence
```

```
    the statistics of records but
    it is useful to adjust the
    random series to resemble the
    temperature record in deg C *)

  a[[i]] = a[[i]] * damping;
  (* But the temperature anomaly is
     shrunk by a factor to avoid
     sqrt(time)-like Brownian
     runaway behavior. Physically,
  this line gives the climate
    the effects that stabilize the
    temperature if it deviates too
    much. Any factor below 1 keeps
    the temperature oscillating
    in a finite range around zero.
     Actually, 0.99 is completely
       tolerable and doesn't increase
       the standard deviation of
       temperature over very long
```

```
      time scales too much...
    In fact, the long-
    term standard deviation of
     temperature seems to scale
     like 1/sqrt(1-damping),
  as I determined by games
    below the memo.
     It follows that for 129 years,
  damping can be so close to one
    that we may pretty much say
    that the temperature dynamics
    is essentially Brownian motion
  *)

];
(* Dynamic[
  ListLinePlot[a,Filling→Axis]] *)

If[a[[t]] < 0, a = -a, a = a];
recordYearIDs = Ordering[
```

```
    -a, recordYearsLookedFor];
  yesNoRecent = If[# > t - whereAllowed,
      1, 0] & /@ recordYearIDs;
  howManyRecent = Total[yesNoRecent];
  alarmistPlanets =
   alarmistPlanets + If[howManyRecent ==
      recordYearsLookedFor, 1, 0];
 ];

{alarmistPlanets,
 N[alarmistPlanets / planets]}
(* probability that we get the
 desired recent record breakers *)
```

```
{127, 13, 17, 0.99}
```

```
{329, 0.0658}
```

```
(* For 129 years and 15 record breakers required to fall into
 the most recent 19 years, the probability is around 6%.

  For 127 years and 13 record breakers
 required to fall to the most recent 15 years,
the probability is also close to 6%.

   This is for damping=0.99 which is a realistic value
  (the standard deviation is about 10 times larger than the
     maximum random increments). For damping at 0.999 or higher
  (which is actually claimed to be relevant in the newer,
   normally distributed notebook),
both figures get close to 10% or slightly above. Further
  increases of damping towards 1 have almost no effect.

   Adding non-Brownian, random noise (without inertia),
like the noise observed at local stations or (partly) regional
 scales, would clearly make the years more independent,
and it would reduce the predicted probability that
 the 13 or so record breakers appear recently. *)
```

Between 5% and 10% of the planets observe 13 record-breaking years (either hottest or coldest) in the most recent 17 years out of 127 on their record.

And that’s the memo.

```
(* What follows are various tests to learn
 how the parameters influence the series *)

{t, damping} = {127 000, 0.99}
a = Table[0, {i, t}]; b = a; c = a;  (* a[[1]] is 1880,
assumed to have temperature anomaly of zero *)

For [i = 2, i ≤ t, i++,
  a[[i]] = a[[i - 1]] + (RandomReal[] - 0.5) * 1;
  (* This adds a Brownian motion-
   like noise to the previous year,
  to derive the new temperature. The overall normalization,
  determined by "times one" at the end,
  doesn’t influence the statistics of records
   but it is useful to adjust the random series
   to resemble the temperature record in deg C *)

  a[[i]] = a[[i]] * damping;
  (* But the temperature anomaly is shrunk by a factor to avoid
    sqrt(time)-like Brownian runaway behavior. Physically,
  this line gives the climate the effects that
   stabilize the temperature if it deviates too
   much. Any factor below 1 keeps the temperature
   oscillating in a finite range around zero.
    Actually, 0.999 is tolerable and doesn’t
    increase the standard deviation of temperature
    over very long time scales too much... *)

 ];
(* ListPlot[a] *)
{  "Standard deviation", Sqrt[ Total[a^2] / t ] ,
 "Theoretical formula for the SD", 0.2 / Sqrt[1 - damping]  }
```

```
{127 000, 0.99}
```

```
{Standard deviation, 2.0533, Theoretical formula for the SD, 2.}
```

```
(* Yes, I tried to change damping from 0.999 to 0.99999,
i.e. the deviation from 1 decreases 100 times,
and the standard deviation for a long-term dynamics,
determined by "t" of order tens of thousands of years,
only grew about 10 times. Try it. *)
```